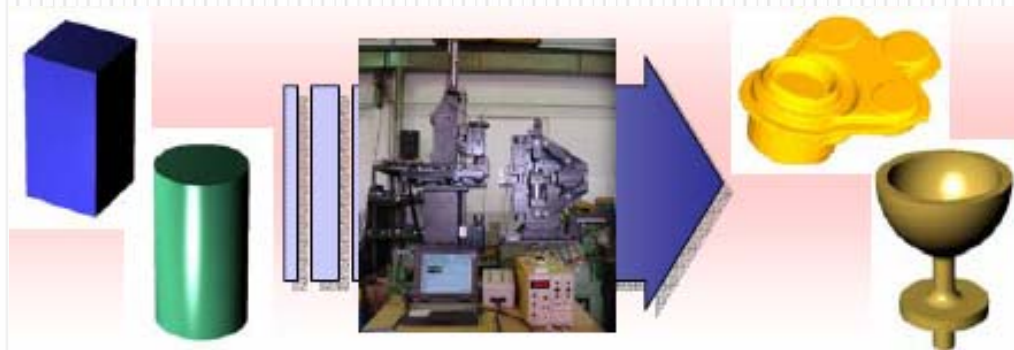


# *NAPREDNE METODE TEHNOLOGIJE PLASTIČNOG DEFORMISANJA*

dr Mladomir Milutinović, vanredni profesor  
dr Marko Vilotić, docent



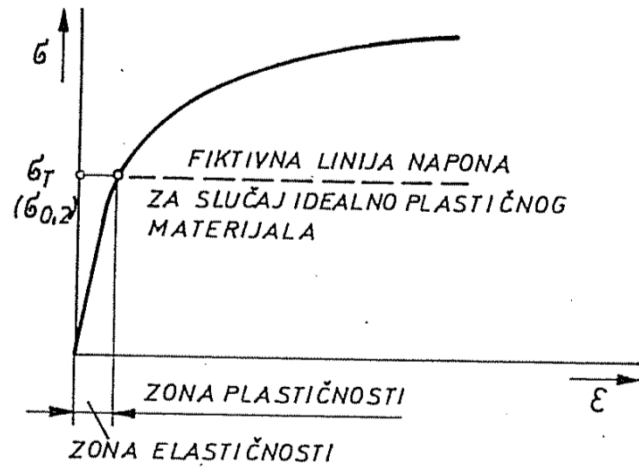
# Uslovi plastičnosti

**Napon plastičnog tečenja** - pokazatelj koji definiše prelazak materijala u oblast plastičnosti.

**Specifični deformacioni otpor (K)** – napon koji pri jednoosnom naponskom stanju dovodi do plastičnog tečenja.

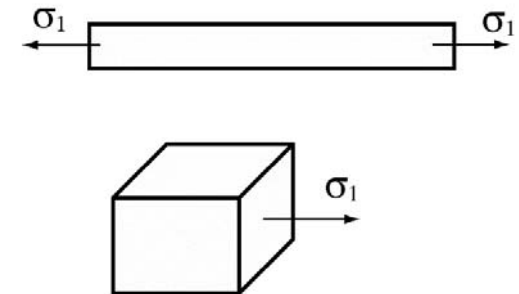
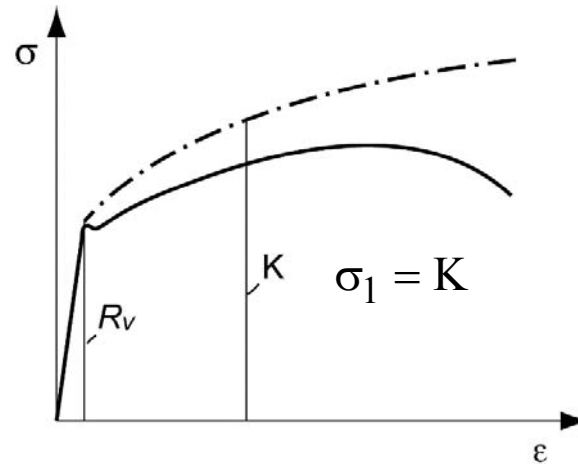
Faktori za prelazak tela u stanje plastičnosti:

- naponsko stanje
- mehanička svojstva materijala



$$\sigma_1 = \sigma_T \approx \sigma_{0.2}$$

$$\sigma_2 = \sigma_3 = 0$$



Dvoosno (ravansko), troosno (prostorno) naponsko stanje????

- **Hipoteza maksimalnog tangencijalnog napona**
- **Energetska hipoteza**

# Hipoteza maksimalnog tangencijalnog napona (Tresca-in uslov plastičnog tečenja)

Prema ovoj hipotezi, plastično deformisanje nastaje onda kada maksimalni tangencijalni napon ( $\tau_{\max}$ ) dostigne određenu kritičnu vrednost.

Ova vrednost zavisi samo od **svojstava materijala i uslova deformisanja** ( $T, \epsilon, \varphi$ )

$$|\tau_{\max}|^{**} = \text{const.}$$

**Henri Tresca**



**Born** 12 October 1814  
Dunkirk

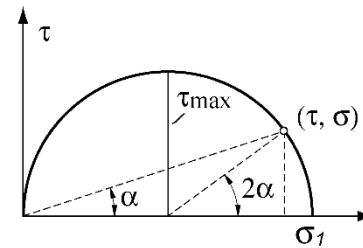
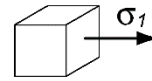
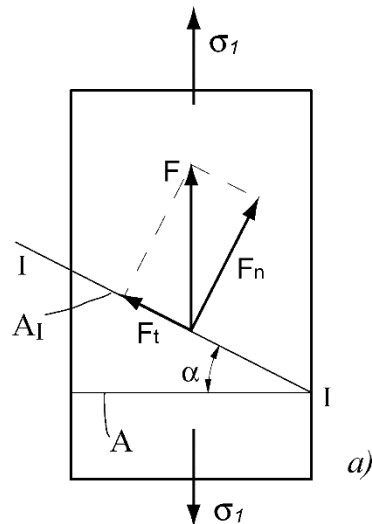
**Died** 21 June 1885 (aged 70)  
Paris

**Nationality** French

**Occupation** Engineer  
Engineering career

**Discipline** Mechanical

**Institutions** American Society of Mechanical Engineers, French Academy of Sciences



• *Jednoosno naponsko stanje*

$$\tau = \frac{F_t}{A_I} = \frac{F \cdot \sin \alpha}{\frac{A}{\cos \alpha}} = \frac{F}{A} \sin \alpha \cdot \cos \alpha = \frac{\sigma}{2} \sin 2\alpha \quad \alpha = 45^\circ \Rightarrow \tau = \tau_{\max}$$

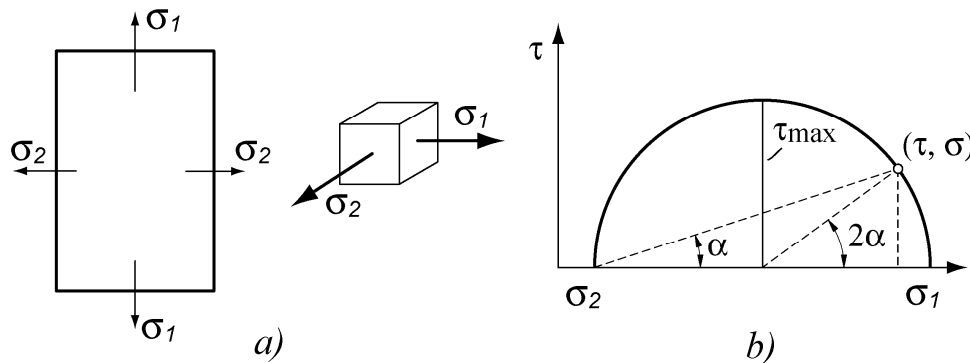
$$\sigma_1 = K$$

$$\tau_{\max} = \frac{\sigma_1}{2}$$

$$\tau_{\max} = \frac{K}{2}$$

# Hipoteza maksimalnog tangencijalnog napona (Tresca-in uslov plastičnog tečenja)

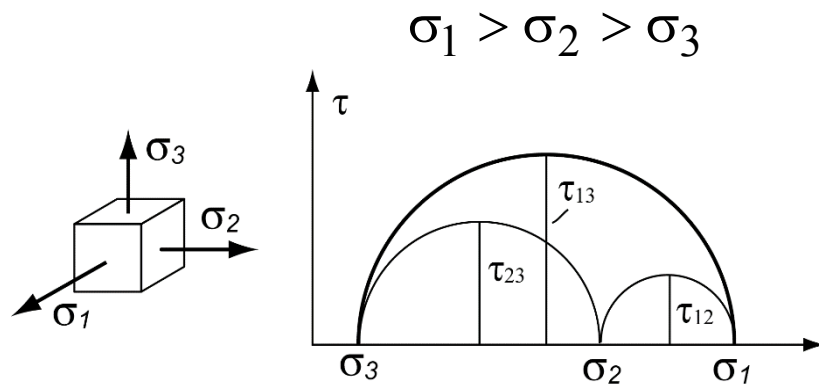
## • Dvoosno naponsko stanje



$$\tau_{\max} = \frac{(\sigma_1 - \sigma_2)}{2} \quad \left( \tau_{\max} = \frac{K}{2} \right)$$

$$\sigma_1 - \sigma_2 = K$$

## • Troosno naponsko stanje



$$\tau_{31} = \frac{1}{2} (\sigma_1 - \sigma_3) = \tau_{\max}$$

$$\sigma_1 - \sigma_3 = K$$

# Energetska hipoteza – Missesov kriterijum

(Hubert-Hencky-Misses-ov kriterijum)

Richard von Mises



Born	19 April 1883 Lemberg, Austria-Hungary (now Lviv, Ukraine)
Died	14 July 1953 (aged 70) Boston, Massachusetts
Known for	von Mises distribution Von Mises transformation Von Mises statistic Bernstein–von Mises theorem Cramér–von Mises criterion von Mises yield criterion Von Mises–Fisher distribution Bivariate von Mises distribution
	<b>Scientific career</b>
Fields	Solid mechanics, fluid mechanics, aerodynamics, aeronautics, statistics and probability theory
Doctoral advisor	Georg Hamel
Doctoral students	Stefan Bergman, Hermine Agavni Kalustyan

***Prema energetske hipotezi, plastična deformacija nastaje kada energija unutrašnje elastične promene oblika (specifična energija) dostigne određenu konstantnu veličinu – tj. dostigne energiju promene oblika pri jednoosnom naponskom stanju.***

***Veličina ove energije ne zavisi od naponsko-deformacione šeme već od svojstava materijala pri dati uslovima deformisanja ( $T, \varepsilon, \varphi$ )***

***Energija elastične promene oblika može rasti samo do određene (kritične) vrednosti, nakon čega dolazi do plastičnog deformisanja (reasterećenja).***

## Energetska hipoteza – Missesov kriterijum

Ukupna energija pri deformaciji tela ( $W_u$ ) troši se na promenu oblika ( $W_o$ ) i promenu njegove zapremine ( $W_v$ ):

$$W_u = W_o + W_v$$

$$W_u = \frac{1}{2}(\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3)$$

$$W_u = \frac{1}{2E} \left[ \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1) \right]$$

$$\varepsilon_1 = \frac{1}{E} \left[ \sigma_1 - \mu(\sigma_2 + \sigma_3) \right]$$

$$\varepsilon_2 = \frac{1}{E} \left[ \sigma_2 - \mu(\sigma_3 + \sigma_1) \right]$$

$$w_o = w - w_v = \frac{1 + \mu_p}{6E} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]$$

$$\varepsilon_3 = \frac{1}{E} \left[ \sigma_3 - \mu(\sigma_1 + \sigma_2) \right]$$

$$W_v = \frac{1}{2}(\sigma_m \varepsilon_m + \sigma_m \varepsilon_m + \sigma_m \varepsilon_m) = \frac{3}{2} \sigma_m \varepsilon_m$$

$$W_v = \frac{1}{2} \Delta V \cdot \sigma_{sr} = \frac{1}{6E} (1 - 2\mu) (\sigma_1 + \sigma_2 + \sigma_3)^2$$

$$W_o = W_u - W_v = \frac{1 + \mu}{6E} (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

## Energetska hipoteza – Missesov kriterijum

$$W_o = W_u - W_v = \frac{1+\mu}{6E} (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

Jednoosno naponsko stanje (granica tečenja)

$$\sigma_1 \neq 0, \quad \sigma_2 = \sigma_3 = 0 \quad \sigma_e = \sigma_1 = K$$

$$W_{oT} = \frac{1+\mu}{2E} K^2 = \text{const.}$$

Opšti analitički izraz za energetski uslov tečenja (efektivni napon)

$$\sigma_e = \frac{\sqrt{2}}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sigma_{ek} = K$$

$$\sigma_e = \frac{\sqrt{2}}{2} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} = \sigma_{ek} = K$$

# Poređenje dva kriterijuma

Troosno naponsko stanje

$$\sigma_1 = \sigma_2 \text{ ili } \sigma_2 = \sigma_3 \rightarrow \sigma_1 - \sigma_3 = K$$

Ekstremni slučaj (ravansko deformaciono stanje)

$$\sigma_2 = \frac{1}{2}(\sigma_1 + \sigma_3) \rightarrow \sigma_1 - \sigma_3 = \frac{2}{\sqrt{3}} K \approx 1,15 \cdot K$$

$$(\varepsilon_1 = -\varepsilon_3, \varepsilon_2 = 0)$$

$$\tau_{\max} = \frac{K}{\sqrt{3}} = 0,578K$$

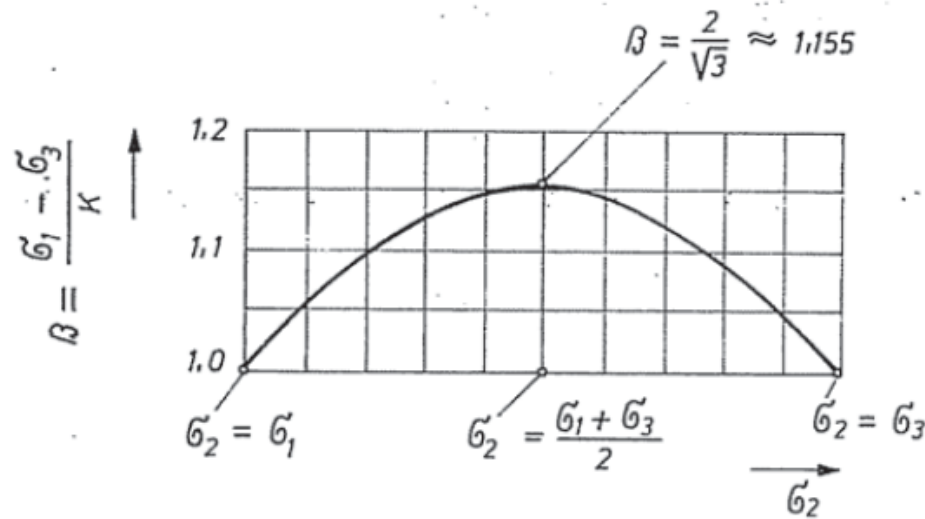
Opšti slučaj

$$\sigma_{\max} - \sigma_{\min} = \beta \cdot K$$

$$\tau_{\max} = \beta \cdot K$$

$$1 < \beta < \frac{2}{\sqrt{3}} = 1,15$$

$\beta$  - Lode-ov koeficijent



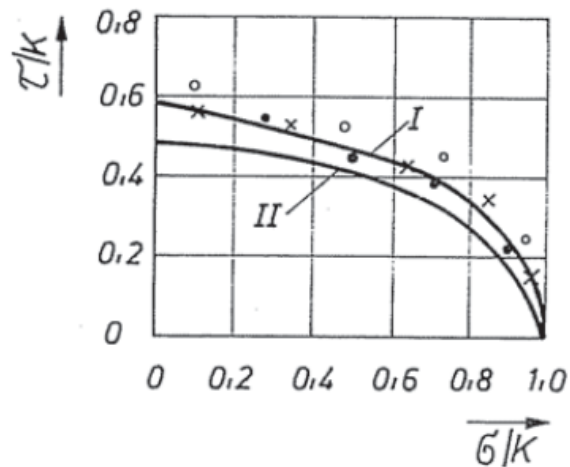


## Poređenje dva kriterijuma

$$\sigma_{\max} - \sigma_{\min} = \beta \cdot K$$

$$\tau_{\max} = \beta \cdot K$$

$$1 < \beta < \frac{2}{\sqrt{3}} = 1,15$$



○ — MEKI ČELIK  
 • — BAKAR  
 × — ALUMINIJUM

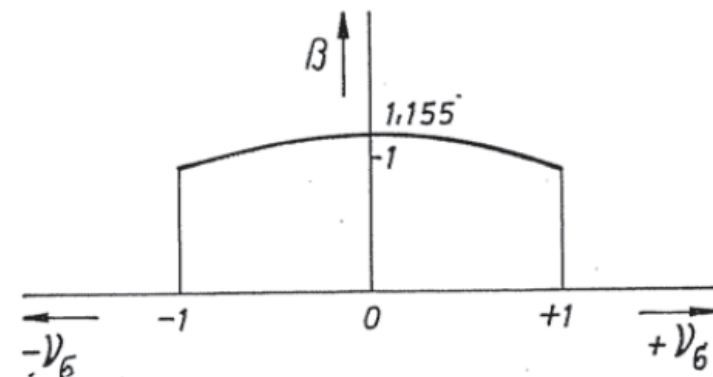
USLOVI PLASTIČNOSTI

I — v. Mises  
 II — Tresca

$$\sigma_2 = \frac{\nu_{\sigma} (\sigma_1 - \sigma_3)}{2} + \frac{\sigma_1 + \sigma_3}{2}$$

$$\beta = \frac{2}{\sqrt{3 + \nu_{\sigma}^2}}$$

$$(\pm \sigma_1) - (\pm \sigma_3) = \beta K$$

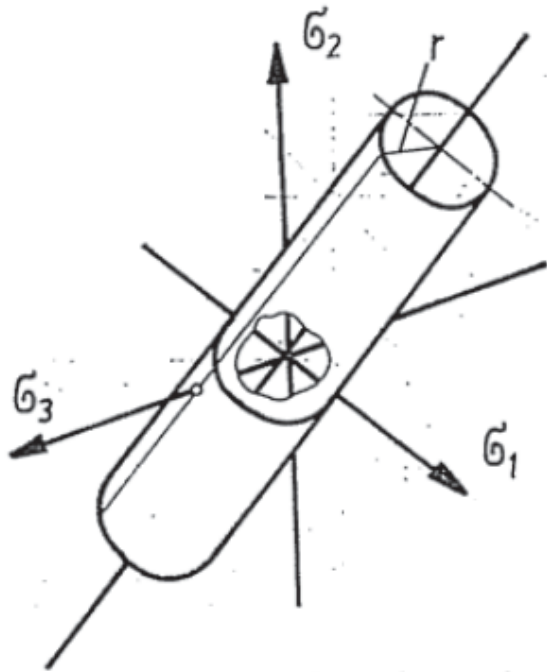


# Geometrijska interpretacija uslova plastičnog tečenja

Mises-ov kriterijum

$$2K^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

jednačina površine cilindra  $r = \sqrt{\frac{2}{3}}K$



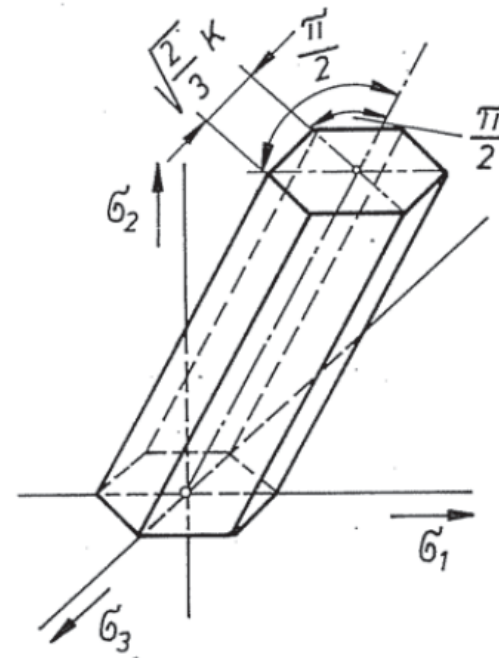
Tresca-in kriterijum

$$|\sigma_1 - \sigma_2| \leq K^*$$

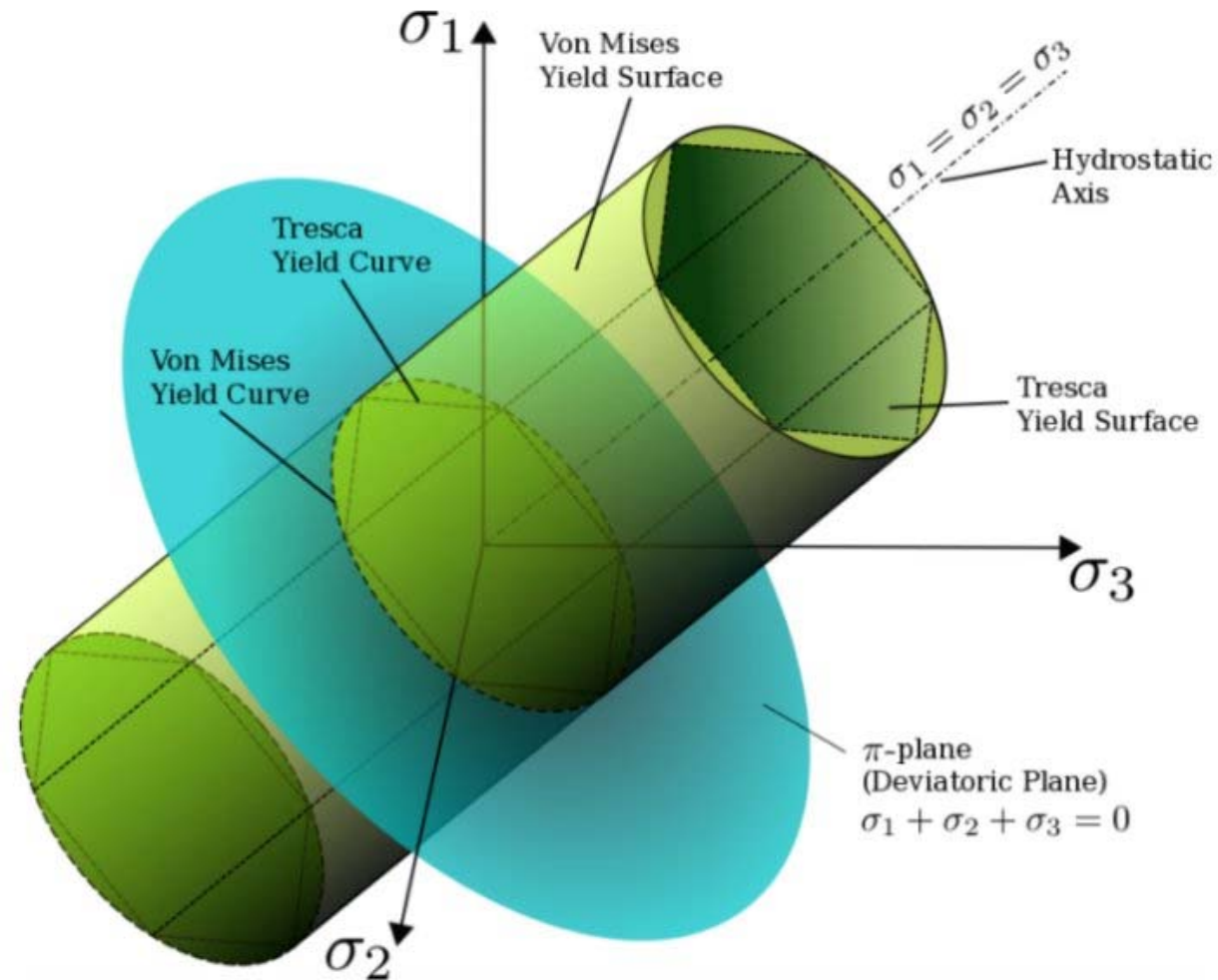
$$|\sigma_2 - \sigma_3| \leq K^*$$

$$|\sigma_3 - \sigma_1| = K$$

šestostrana prizma  $\sqrt{\frac{2}{3}}K$



# Geometrijska interpretacija uslova plastičnog tečenja



# Geometrijska interpretacija uslova plastičnog tečenja

Ravansko naponsko stanje

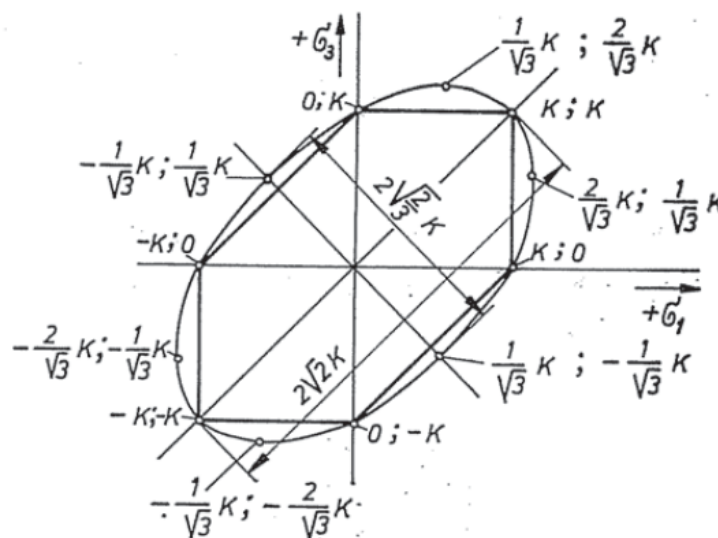
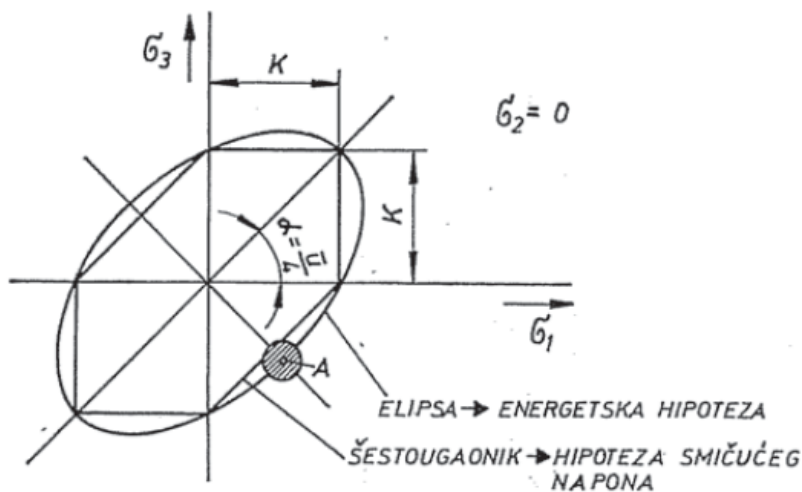
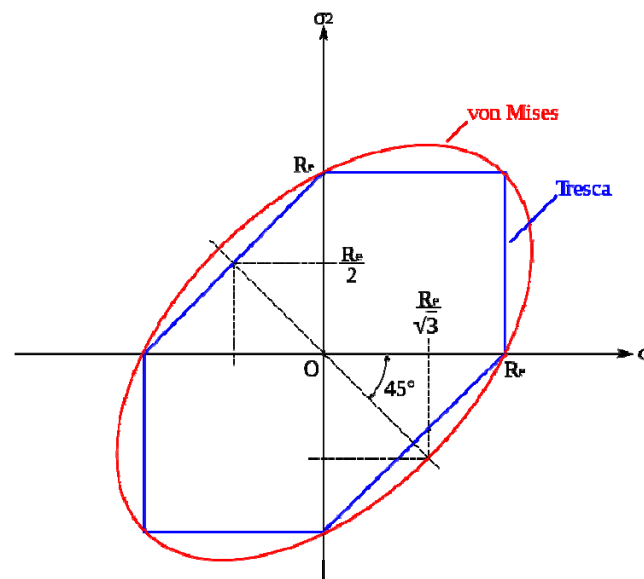
$$\sigma_2^2 + \sigma_3^2 - \sigma_2 \sigma_3 = K^2$$

$$\sigma_1^2 + \sigma_3^2 - \sigma_1 \sigma_3 = K^2$$

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = K^2$$

jednačine elipsi

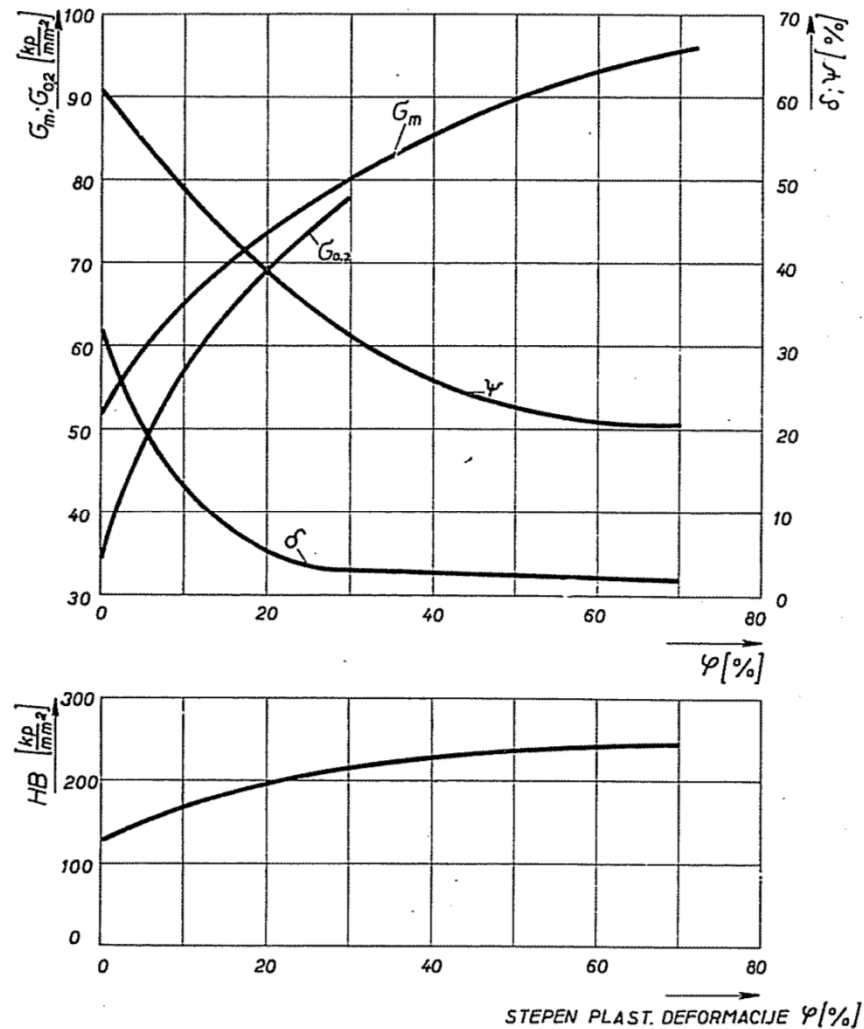
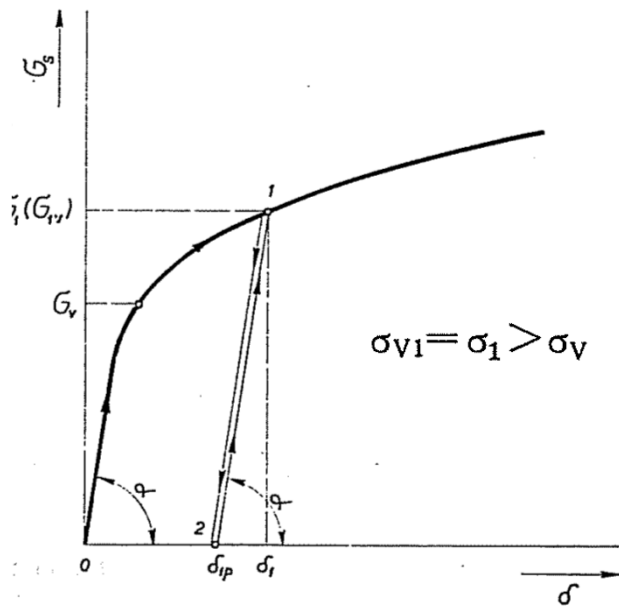
$$2/\sqrt{3} K = 1,155 K$$



# Krive tečenja

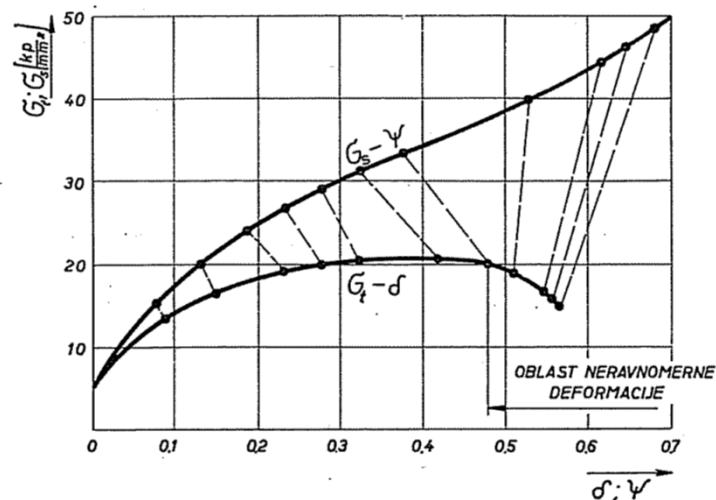
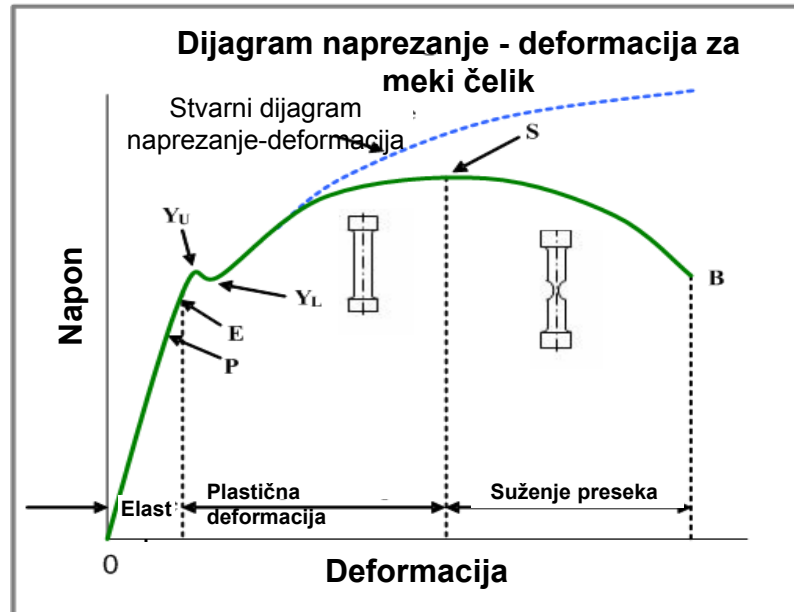
Pri deformisanje metala u hladnom stanju dolazi do promene svojstva metala i vrednosti mehaničkih karakteristika/pokazatelja ( $\sigma$ , HB,  $\psi$ ,  $\delta$ ...).

Deformaciono ojačavanje  
Deformacioni otpor



# Krive tečenja

Stvarni i nominalni (tehnički) napon



**Nominalni napon**

$$\sigma = \frac{F}{A_0}$$

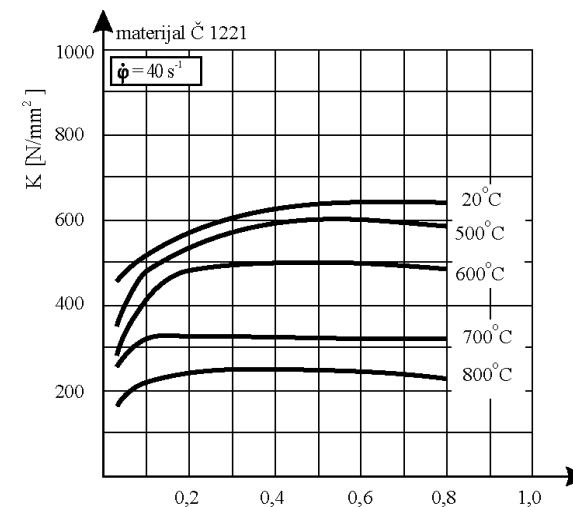
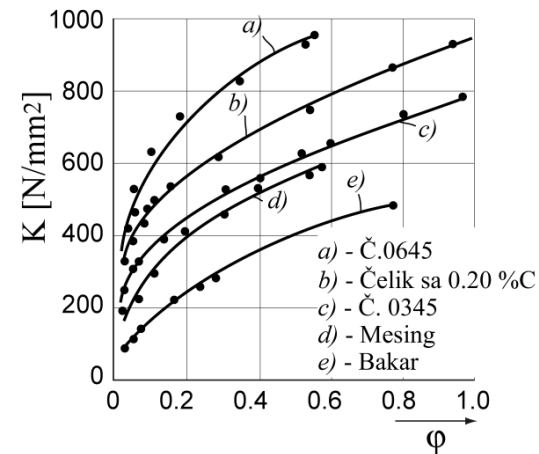
$$R_e = \frac{F_E}{A_0} ; R_m = \frac{F_M}{A_0}$$

**Stvarni napon**

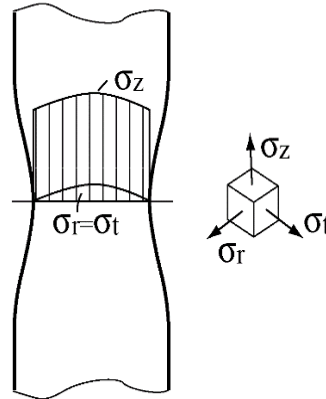
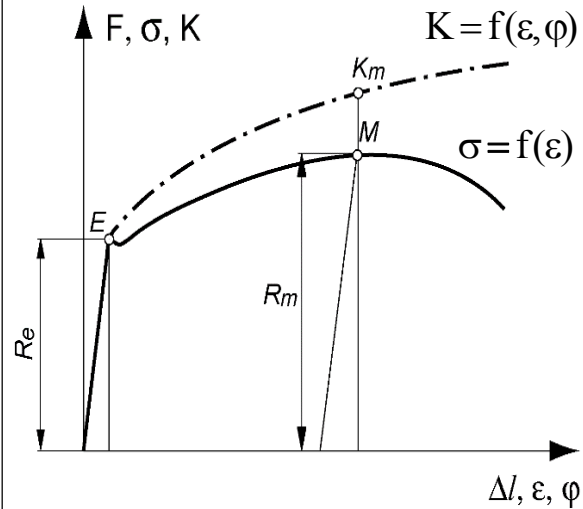
$$K = \frac{F}{A} = \sigma_e$$

# Krive tečenja

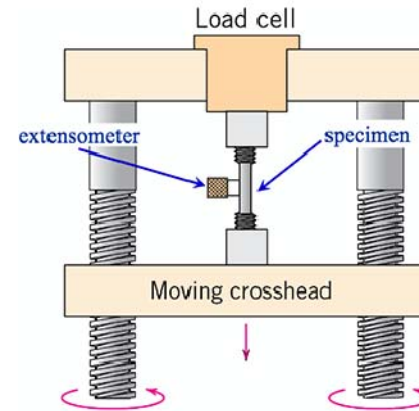
- ❑ Napon tečenja (**K**) predstavlja vrednost normalnog napona u trenutku pojave plastične deformacije. Napon tečenja se menja sa deformacijom!!
- ❑ Kriva tečenja predstavlja funkcionalnu zavisnost napona tečenja (**K**) i stepena logaritamske deformacije ( $\phi$ ).
- ❑ Kriva tečenja (realna) zavisi od:
  - vrste materijala
  - polazne strukture
  - temperature obrade
  - brzine deformacije.
- ❑ Eksperimentalno određivanje
- ❑ Metode određivanja krive tečenja:
  1. Jednoosno zatezanje
  2. Dvoosno zatezanje
  3. Sabijanje
  4. Uvijanje (torzija)



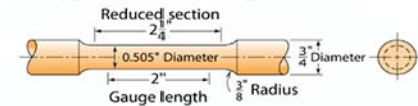
# Određivanja krive tečenja jednoosnim zatezanjem



Typical tensile test machine



Typical tensile specimen



$$K = \frac{F}{A} = \frac{F}{A} \frac{A_0}{A_0} = \sigma \frac{A_0}{A}$$

$$A_0 \cdot l_0 = A \cdot l \rightarrow \frac{A_0}{A} = \frac{l}{l_0}$$

$$\varepsilon = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1$$

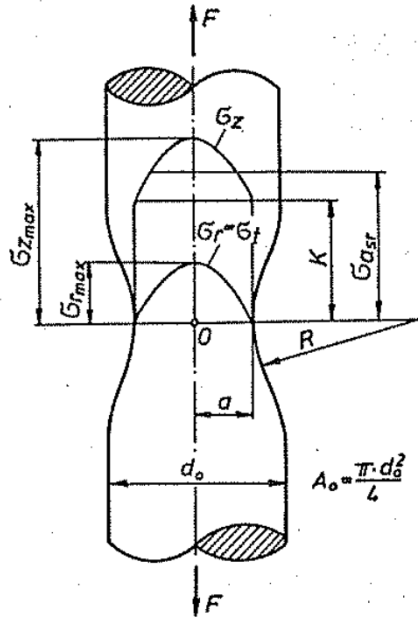
$$K = \sigma(1 + \varepsilon)$$

$$\varphi_r = \varphi_m = \ln \frac{l_m}{l_0} = \varphi_e < 0,3$$





# Određivanja krive tečenja jednoosnim zatezanjem



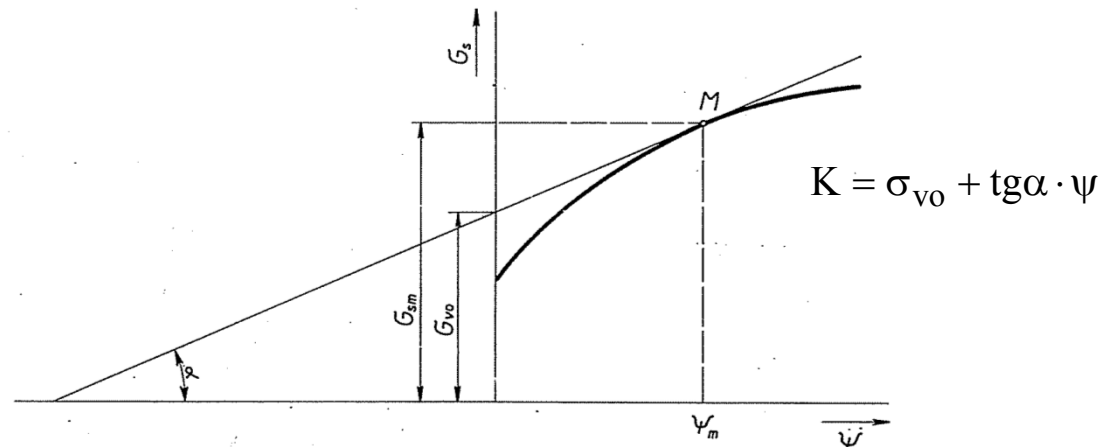
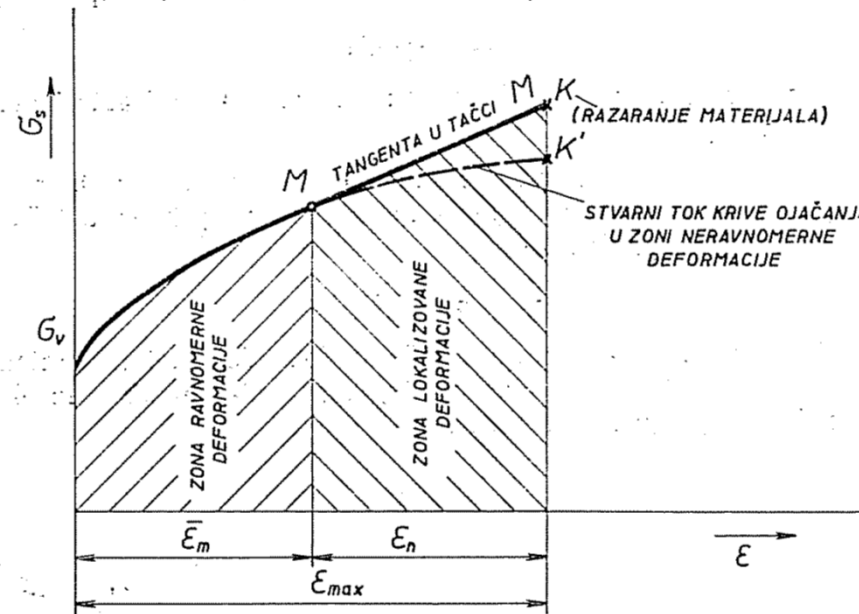
$$\sigma_r = \sigma_t = K \cdot \ln \left[ \frac{a^2 + 2a \cdot R - r^2}{2 \cdot a \cdot r} \right]$$

$$\sigma_z = K \cdot \ln \left[ \frac{a^2 + 2aR - r^2}{2aR} \right] + K$$

$$K = \sigma_z - \sigma_r$$

$$K = \frac{\sigma_{asr}}{\left[ 1 + 2 \left( \frac{R}{a} \right) \right] \cdot \ln \left[ 1 + \frac{a}{2R} \right]}$$

$$\sigma_{asr} = \frac{F}{\pi \cdot a^2}$$



# Određivanja krive tečenja jednoosnim zatezanjem

## Analitička aproksimacija krive tečenja

$$K = C \cdot \varphi_e^n$$

Određivanje konstante **C** i eksponenta deformacionog ojačavanja **n** vrši se na osnovu podataka dobijenih ispitivanjem na zatezanje. Ove konstante zavise od vrste materijala.

$$F = A \cdot K = A \cdot C \cdot \varphi_e^n$$

$$\varphi_e = \ln \frac{l}{l_0}, \quad l_0 \cdot A_0 = l \cdot A \Rightarrow A = A_0 \cdot e^{-\varphi_e}$$



$$F = C \cdot \varphi_e^n \cdot A_0 \cdot e^{-\varphi_e}$$

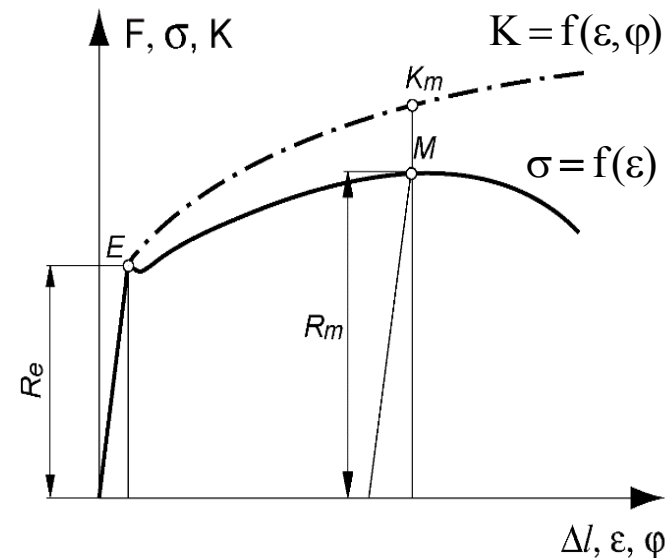
$$\frac{dF}{d\varphi_e} = C \cdot A_0 \left( n \cdot \varphi_e^{n-1} \cdot e^{-\varphi_e} - \varphi_e^n \cdot e^{-\varphi_e} \right) = 0$$

$$\varphi_e = \varphi_m \Rightarrow n \cdot \varphi_{em}^{n-1} - \varphi_{em}^n = 0$$

$$n = \varphi_{em} = \varphi_r$$

$$K_m = C \cdot \varphi_{em}^n = C \cdot n^n$$

$$C = \frac{K_m}{n^n}$$



# Analitička aproksimacija krive tečenja

Postoji više od 20 različitih modela za aproksimaciju krivi tečenja!!!

$$K = C \varphi^n \quad \text{Hollomon (power law curve, 1944)}$$

$$K = K_0 + C \cdot \varphi^n \quad \text{Ludwig (1909)}$$

$$K = K_0 + (B + \varphi)^n$$

$$K = A (B + \varphi)^n \quad \text{Swift}$$

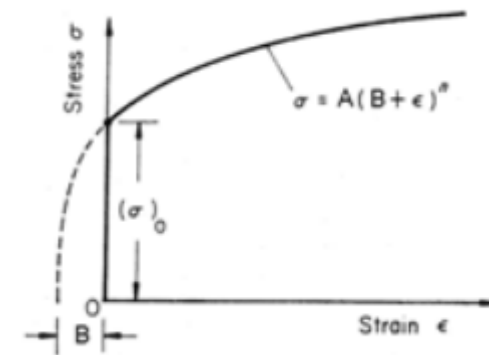
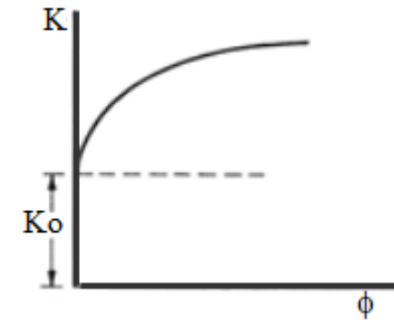
$$K = K_0 + B_1 \cdot \log \varphi$$

$$K = K_0 + B_2 [1 - \exp(-C \cdot \varphi)]^n$$

$$K = A_t \varepsilon^n \dot{\varepsilon}^m$$

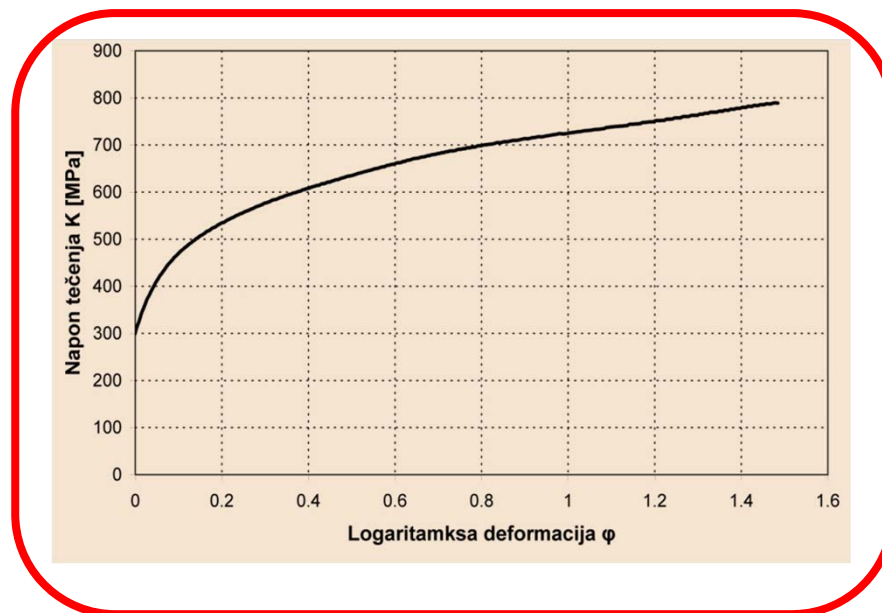
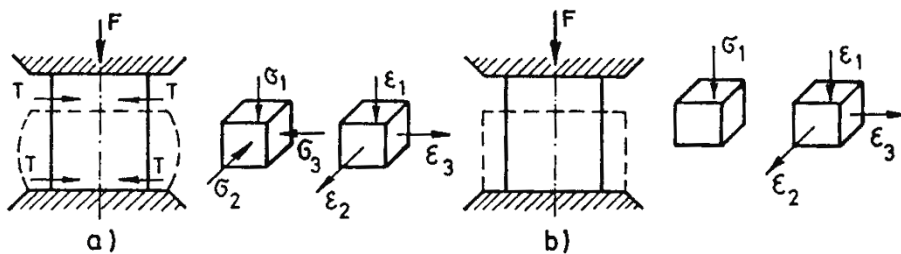
$$K = A_3 \varepsilon^n (A_4 + A_5 \ln \dot{\varepsilon})$$

} Topla obrada



# Određivanja krive tečenja sabijanjem

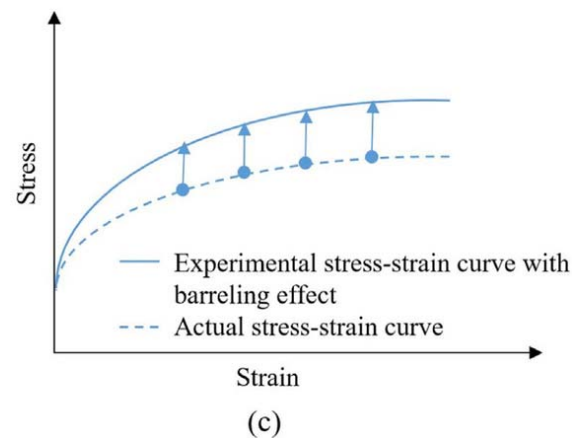
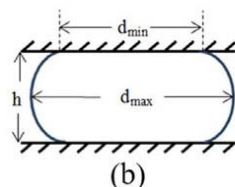
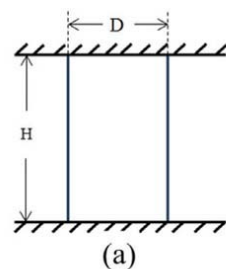
- Koristi se za određivanje krive tečenja u širem opsegu deformacija ( $\varphi > 1$ )
- Jednostavan i brz test



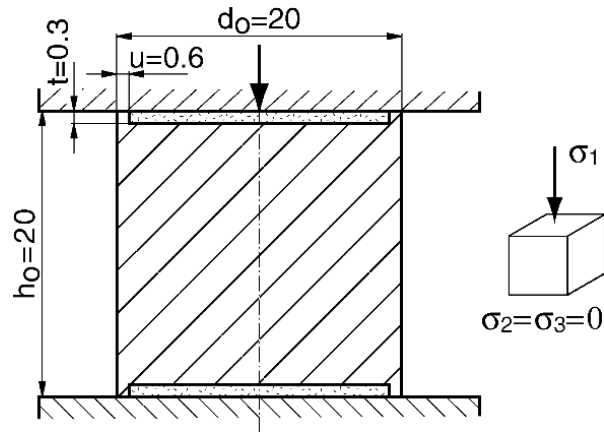
$$F = K \cdot A \left( 1 + \frac{\mu d}{3 h} \right)$$

$$K = \frac{F}{A \left( 1 + \frac{\mu d}{3 h} \right)}$$

$$\varphi = \ln \frac{h_0}{h}$$



# Određivanja krive tečenja jedoosnim sabijanjem metoda Rastegajeva



Za čelike:

$d_0 = 7 \div 20$  mm, širina oboda  $u = 0.45 \div 0.6$  mm, i dubina  $t = 0.3 \div 0.4$  mm,

Za meke metale (Cu, Al):

$u = 1.7 \div 2.3$  mm i  $t = 0.6 \div 0.65$  mm

Podmazivanje: **stearin**

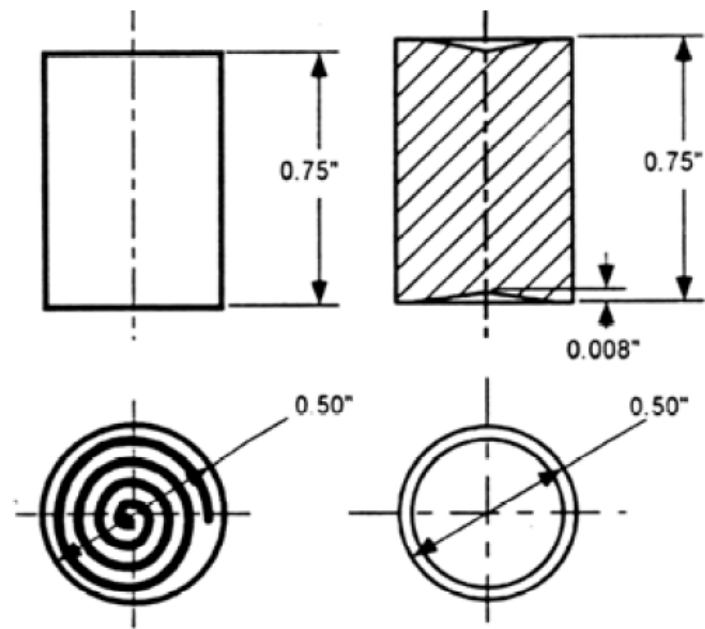
$$\sigma_1 = K = \frac{F}{A}$$

$$|\varphi_n| = \varphi_e = \ln \frac{h_0}{h} = \ln \frac{A}{A_0}$$

$$\varphi_{\max} = 1,2 \div 1,4$$

$D_0 = h_0$	$u_0$	$t_0$
10	0.3	0.15
14	0.4	0.2
20	0.5	0.4

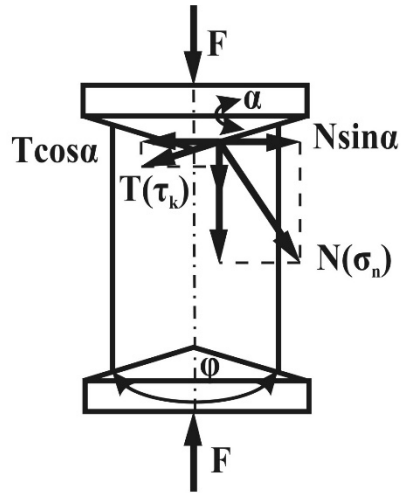
## Oodređivanja krive tečenja jedoosnim sabijanjem



Spiralni žleb:  $\varphi_{\max} = 0,6 \div 0,7$   
Konusno udubljenje:  $\varphi_{\max} \approx 2$



# Određivanja krive tečenja jedoosnim sabijanjem



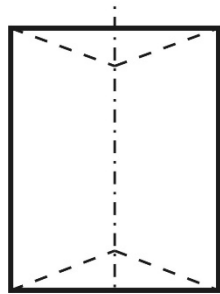
$$\mu = \operatorname{tg} \alpha$$

$$K = \frac{F}{A} = \frac{F}{\frac{D^2 \pi}{4}}$$

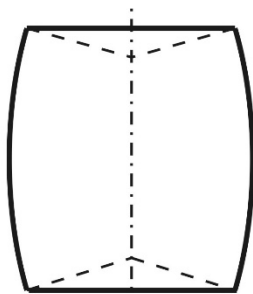
$$T = N \operatorname{tg} \alpha$$

$$T > N \operatorname{tg} \alpha$$

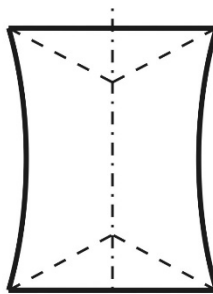
$$T < N \operatorname{tg} \alpha$$



$$\mu = \operatorname{tg} \alpha$$



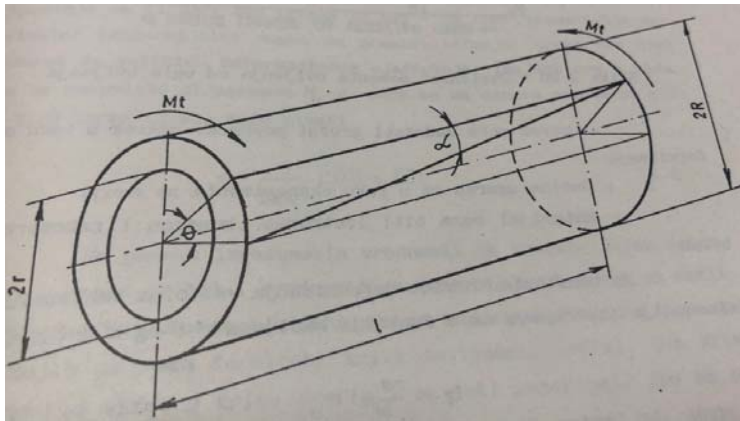
$$\mu > \operatorname{tg} \alpha$$



$$\mu < \operatorname{tg} \alpha$$

## Određivanja krive uvijanjem (torzija)

- Omogućuje određivanje napona tečenja do velikih iznosa deformacija  $\varphi_{\max} = 2 \div 4$ , ali zahteva specijalnu opremu.
- Uzorak bitno ne menja oblik i nema nestabilnosti procesa (lokalizacija, buričenje)
- Tačniji rezultati u odnosu na test sabijanja (nema trenja)
- Nije standardizovan
- Složena obrada podataka eksperimenta (promena deformacije i brzine deformisanja)
- Postupak se sastoji iz tri faze:
  1. Uspostavlja se veza između momenta uvijanja ( $M_t$ ) i ugla uvijanja ( $\theta$ )
  2. Izračunava se napon smicanja ( $\tau$ ) i ugaona (smičuća) deformacija ( $\gamma$ )





## Određivanja krive uvijanjem (torzija)

$$\gamma = \frac{r \cdot \theta}{l}$$

$$M_t = \frac{2\pi}{\theta^3} \int_0^R \tau \cdot \gamma d\gamma$$

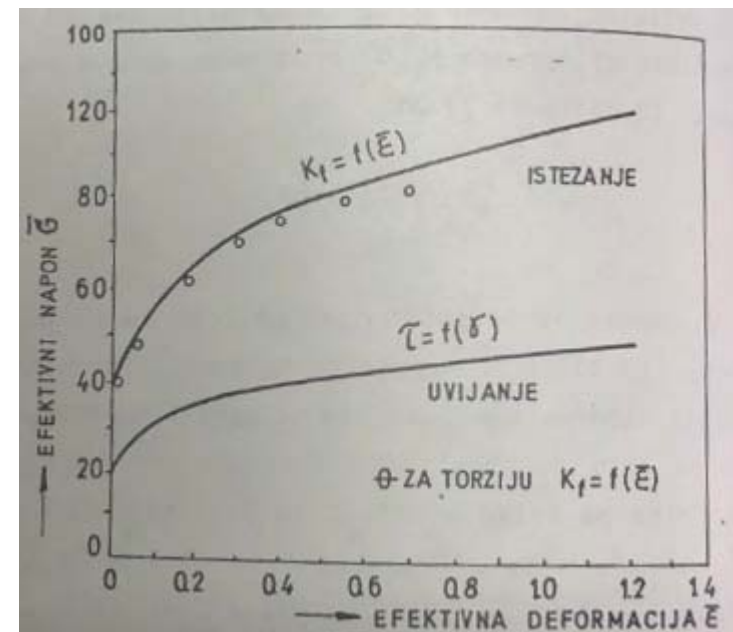
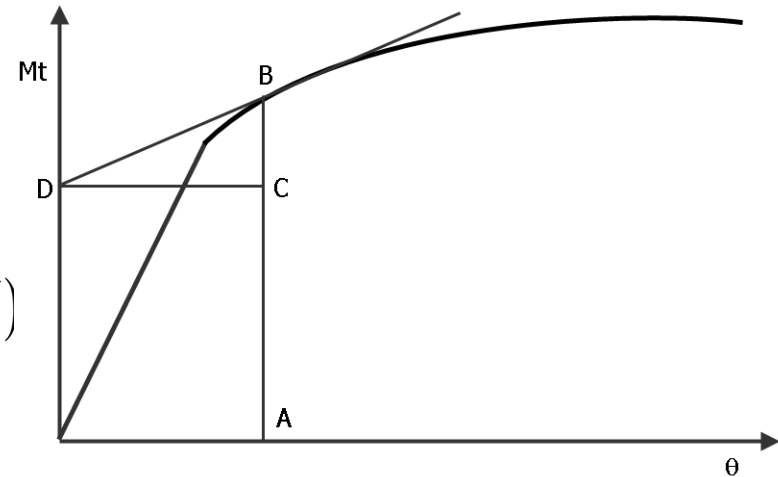
$$\tau_R = \frac{1}{2\pi \cdot R^3} \left( 3M_t + \frac{dM_t}{d\theta} \right) \quad \tau_R = \frac{1}{2\pi \cdot R^3} (3\overline{AB} + \overline{BC})$$

$$K = \sqrt{3}\tau_e$$


$$\varphi_e = \frac{\varphi}{\sqrt{3}} = \frac{r \cdot \theta}{\sqrt{3} \cdot l}$$

Razlike:

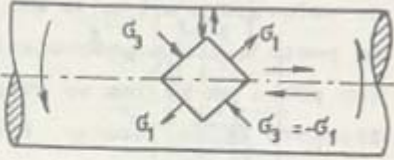
1. Različiti kriterijumi tečenja materijala
2. Materijal ostaje izotropan (teksture deformacije se razlikuju)
3. Brzina deformacionog ojačavanja



# Određivanja krive uvijanjem (torzija)



Test rastezanja



Torzioni test

$\sigma_1 = \sigma_{\max}; \sigma_2 = \sigma_3 = 0$ $\tau_{\max} = \frac{\sigma_1}{2} = \frac{\sigma_{\max}}{2}$ $\varphi_{\max} = \varphi_1; \varphi_2 = \varphi_3 = -\frac{\varphi_1}{2}$ $\gamma_{\max} = \sin h \frac{3\varphi_1}{2}$	$\sigma_1 = -\sigma_3; \sigma_2 = 0$ $\tau_{\max} = \frac{2\sigma_1}{2} = \sigma_1$ $\varphi_{\max} = \varphi_1 = -\varphi_3; \varphi_2 = 0$ $\gamma_{\max} = \varphi_1 - \varphi_3 = 2\varphi_1$
$k_f = \frac{\sqrt{2}}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$	
$\bar{c} = \left[ \frac{2}{3} (\varphi_1^2 + \varphi_2^2 + \varphi_3^2) \right]^{1/2}$	
$k_f = \sigma_1$ $\bar{c} = \varphi_1$	$k_f = \sqrt{3} \sigma_1$ $\bar{c} = \frac{2}{\sqrt{3}} \varphi_1 = \frac{\gamma}{\sqrt{3}} = \frac{r\theta}{\sqrt{3} l}$